Numeric Response Questions

Indefinite Integration

Q.1 If $\int \frac{1}{x\sqrt{1-x^3}} \mathrm{dx} = \mathrm{a}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + b$, then find value of a.

Q.2 If $\int \frac{\cos 4x+1}{\cot x-\tan x} dx = -\frac{1}{\lambda}\cos 4x + B$, then find value of λ.

Q.3 If $\int \frac{2x^2+3}{(x^2-1)(x^2-4)} dx = \log\left(\frac{x-2}{x+2}\right)^a\left(\frac{x+1}{x-1}\right)^b + \mathrm{c}$, then find value of a + b.

Q.4 If $\int \mathrm{e}^{3\log x}(x^4+1)^{-1}\mathrm{dx} = \frac{1}{k}\log(x^4+1) + e$ then find value of k.

Q.5 If $\int \mathrm{e}^{x/2}\sin\left(\frac{\mathrm{x}}{2}+\frac{\pi}{4}\right)\mathrm{dx} = ke^{v/2}\sin\frac{\mathrm{x}}{2} + \mathrm{e}$ then find k.

Q.6 If $\int \frac{dx}{2x^2+x+1} = \frac{1}{2}\tan^{-1}\left(\frac{4x+1}{\sqrt{k}}\right) + c$ then find k.

Q.7 If $\int (\sin^4 x - \cos^4 x)dx = -\frac{\sin 2x}{k} + c$ then find k.

Q.8 If $\int \frac{dx}{(2x-7)\sqrt{x^2-7x+12}} = \sec^{-1}(ax-b) + c$ then find $a+b$.

Q.9 If $\int f(x)dx = g(x)$ and $\int x'' f(x^6)dx = \frac{1}{a}x^6 g(x^9) - \int x^b g(x^6)dx + c$ then find $a+b$.

Q.10 If $\int \sqrt{\frac{\cos x-\cos^3 x}{1-\cos^3 x}} dx = k\cos^{-1}(\cos^{42} x) + c$ then find k.

Q.11 If $\int \frac{e^{5\log_6 x}-e^{4\log_0 x}}{\mathrm{e}^{3\log_4 x}-e^{2\log e^x}} dx = \frac{x^3}{\lambda} + \mathrm{c}$, then find λ

Q.12 If $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha} dx = \lambda(\sin x + x\cos\alpha) + \mathrm{c}$ then find λ.

Q.13 If $\int \frac{3x+2}{4x^2+4x+5} dx = p\ln(4x^2+4x+5) + q\tan^{-1}(x+a) + c$, then find $p+q-a$

Q.14 If $\int \frac{\sin^2 x}{\cos^6 x} dx$ is a polynomial in $\tan x$ then find degree of polynomial.

Q.15 If $\int \frac{1+x^2}{\sqrt{1-x^2}} \mathrm{dx} = \mathrm{k}\sin^{-1} x - \lambda x\sqrt{1-x^2} + c$ then find $k+\lambda$

ANSWER KEY

1. 0.33	**2.** 8.00	**3.** 1.75	**4.** 4.00	**5.** 1.41	**6.** 7.00	**7.** 2.00
8. 9.00	**9.** 11.00	**10.** 0.67	**11.** 3.00	**12.** 2.00	**13.** 0.00	**14.** 5.00
15. 2.00						

Hints & Solutions

1. $\int \frac{dx}{x\sqrt{1-x^3}}$

put $1 - x^3 = t^2$

$\Rightarrow -3x^2dx = 2t\,dt \Rightarrow dx = \frac{-2t\,dt}{3x^2}$

$= \int \frac{1}{x\sqrt{t^2}} \times \left(\frac{-2t\,dt}{3x^2}\right)$

$= \frac{2}{3}\int \frac{dt}{(-x^3)}$

$= \frac{2}{3}\int \frac{dt}{t^2-1}$

$= \frac{2}{3}.\frac{1}{2.1}\log\left(\frac{t-1}{t+1}\right) + C$

$a = \frac{1}{3}$

2. $\int \frac{2\cos^2 2x - 1 + 1}{\frac{\cos 2x}{\sin x.\cos x}}dx = \int \sin 2x.\cos 2x\,dx$

$= \frac{1}{2}\int \sin 4x\,dx$

$= -\frac{1}{8}\operatorname{cosec} x + C$

3. $\int \frac{-5/3}{x^2-1} + \frac{11/3}{x^2-4}$

$= \frac{-5}{3}.\frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + \frac{11}{3}.\frac{1}{2.2}\log\left(\frac{x-2}{x+2}\right) + C$

$= \frac{5}{6}\log\left(\frac{x+1}{x-1}\right) + \frac{11}{12}\log\left(\frac{x-2}{x+2}\right) + C$

4. $\int e^{\log x^3}(x^4+1)^{-1}dx$

$= \int \frac{x^3}{x^4+1}dx$ Let $x^4 + 1 = t$

$= \frac{1}{4}\log(x^4+1) + C$

5. $\int e^{\frac{x}{2}}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

$= \int e^{\frac{x}{2}}\frac{1}{\sqrt{2}}\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)dx$

Let $\frac{x}{2} = t \Rightarrow dx = 2dt$

$= \sqrt{2}\int e^t(\sin t + \cos t)dt$

$\because \frac{d}{dt}(\sin t) = \cos t$

$I = \sqrt{2}\,e^t \sin t$

$= \sqrt{2}\,e^{\frac{x}{2}}\sin\frac{x}{2} + c$

6. $I = \frac{1}{2}\int \frac{dx}{x^2 + \frac{x}{2} + \frac{1}{2}}$

$\Rightarrow I = \frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$

$I = \frac{1}{2}.\frac{4}{\sqrt{7}}.\tan^{-1}\left[\frac{x+\frac{1}{4}}{(\sqrt{7}/4)}\right] + c$

$\Rightarrow I = \frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{4x+1}{\sqrt{7}}\right] + c$

7. $\int(\sin^4 x - \cos^4 x)dx$

$= \int(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)dx$

$= \int(\sin^2 x - \cos^2 x)dx$

$= -\int(\cos^2 x - \sin^2 x)dx = -\int \cos 2x\, dx$

$= \frac{-\sin 2x}{2} + c$

9. $I = \int x^{11} f(x^6)dx$

put $x^6 = t$ $\Rightarrow 6x^5\, dx = dt$

$I = \frac{1}{6}\int t\, f(t)dt$

$= \frac{1}{6}\left[t\, g(t) - \int g(t)dt\right]$

$= \frac{1}{6}\left[x^6 g(x^6) - \int g(x^6)d(x^6)\right]$

$= \frac{1}{6}\left[x^6 g(x^6) - 6\int x^5 g(x^6)dx\right]$

$= \frac{1}{6} x^6 g(x^6) - \int x^5 g(x^6)dx + C$

10. $\int \frac{\sqrt{\cos x(1-\cos^2 x)}}{\sqrt{1-\cos^3 x}}dx$

$= \int \frac{\sqrt{\cos x}\sin x\, dx}{\sqrt{1-(\cos^{3/2} x)^2}}dx$

Put $t = \cos^{3/2}x$

$\frac{dt}{dx} = \frac{3}{2}\cos^{1/2}x(-\sin x)$

$-\frac{2}{3}dt = \sqrt{\cos x}\sin x\, dx$

$= -\frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{2}{3}\cos^{-1}(t) + c$

11. $\int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}}dx$

$= \int \frac{x^5 - x^4}{x^3 - x^2}dx$

$= \int \frac{x^2(x^3 - x^2)}{x^3 - x^2}dx$

$= \int x^2\, dx$

$= \frac{x^3}{3} + C$

12. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}dx$

$= \int \frac{2\cos^2 x - 1 - 2\cos^2 \alpha + 1}{\cos x - \cos\alpha}dx$

$= 2\int \frac{\cos^2 x - \cos^2 \alpha}{\cos x - \cos \alpha}dx$

$= 2\int (\cos x + \cos \alpha)\, dx$

$= 2(\sin x + x\cos\alpha) + c$

13. $I = \int \frac{3x+2}{4x^2+4x+5}dx$

$3x + 2 = A(4x^2 + 4x + 5)' + B$

$= A(8x + 4) + B$

Solving this, we get

$A = \frac{3}{8}, B = \frac{1}{2}$

$I = \frac{3}{8}\int \frac{8x+4}{4x^2+4x+5}dx + \frac{1}{2}\int \frac{1}{4x^2+4x+5}$ dx

$= \frac{3}{8}\log(4x^2 + 4x + 5) + \frac{1}{2} \times \frac{1}{4}\tan^{-1}\left(x + \frac{1}{2}\right) + c$

$p = \frac{3}{8}$, $q = \frac{1}{8}$, $a = \frac{1}{2}$

Hence, $p + q - a = \frac{3}{8} + \frac{1}{8} - \frac{1}{2} = 0$

14. $I = \int \frac{\sin^2 x}{\cos^6 x}dx$

$= \int \tan^2 x(1 + \tan^2 x)\sec^2 x\, dx$

$= \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + c$

15. $I = -\int \frac{(1-x^2)-2}{\sqrt{1-x^2}}dx$

$= -\int \sqrt{1-x^2}\, dx + 2\sin^{-1} x + c$

$= \frac{1}{2}\left(-\sqrt{1-x^2}x - \sin^{-1} x\right) + 2\sin^{-1} x + c$

$= \frac{3}{2}\sin^{-1} x - \frac{1}{2}x\sqrt{1-x^2} + c$